Published by Institute of Physics Publishing for SISSA

Received: *December 12, 2006*
Revised: *February 7, 2007*
Accepted: *February 8, 2007*
Published: *February 26, 2007*

Energy loss of gluons, baryons and k-quarks in an $\mathcal{N} = 4$ SYM plasma

Mariano Chernicoff and Alberto Güijosa

Departamento de Física de Altas Energías, Instituto de Ciencias Nucleares
Universidad Nacional Autónoma de México, Apdo. Postal 70-543, México D.F. 04510
E-mail: mariano@nucleares.unam.mx, alberto@nucleares.unam.mx

Abstract: We consider different types of external color sources that move through a strongly-coupled thermal $\mathcal{N} = 4$ super-Yang-Mills plasma, and calculate, via the AdS/CFT correspondence, the dissipative force (or equivalently, the rate of energy loss) they experience. A bound state of k quarks in the totally antisymmetric representation is found to feel a force with a nontrivial k-dependence. Our result for k=1 (or $k = N - 1$) agrees at large N with the one obtained recently by Herzog *et al.* and Gubser, but contains in addition an infinite series of $1/N$ corrections. The baryon ($k = N$) is seen to experience no drag. Finally, a heavy gluon is found to be subject to a force which at large N is twice as large as the one experienced by a heavy quark, in accordance with gauge theory expectations.

Keywords: AdS-CFT Correspondence, Gauge-gravity correspondence.

http://jhep.sissa.it/archive/papers/jhep022007084/jhep022007084.pdf

Contents

1. Introduction and summary

A considerable effort has recently been invested in the study of strongly-coupled thermal non-Abelian plasmas by means of the AdS/CFT correspondence [1−3]. The main motivation is the hope of making contact with experimental data on the strongly-coupled quark-gluon plasma (sQGP) that has been produced at RHIC [4] and will be produced at LHC [5] (for reviews, see, e.g., [6]). At our present stage of knowledge, this will be feasible only if real-world QCD can be reasonably well approximated by at least one of the various 'QCD-like' gauge theories whose dual description is known. In the past few years, encouraging signs in this direction have emerged even for the most rudimentary example [1], SU(N) $\mathcal{N} = 4$ super-Yang-Mills (SYM), which at zero temperature is completely unlike QCD, but at finite temperature is in various respects analogous to deconfined QCD. Indeed, the numerical values of several properties of the sQGP appear to be in the ballpark of the corresponding AdS/CFT predictions for a strongly-coupled SYM plasma, including its strong-to-weak-coupling entropy ratio [7, 8] and its ratio of shear viscosity to entropy density [9, 10].[1] These indications have spurred intense research on various fronts, including attempts to achieve a more realistic model of the sQGP by incorporating the effect of its expansion and/or its finite extent [12−15].

Much of the recent activity in this area has focused on determining the rate at which the thermal non-Abelian plasma dissipates energy. The drag force experienced by a heavy quark that ploughs through a strongly-coupled $\mathcal{N} = 4$ SYM plasma was determined in [16, 17].[2] The closely related heavy quark diffusion coefficient was computed in [19, 16]. Previous related work was carried out in [20]; generalizations can be found in [21−27]. In interesting followup work, the authors of [28−30] studied the profile of the coherent gluonic fields set up by the joint quark-plasma system, which are responsible for taking energy away from the moving quark. Their results appear to be consistent with phenomenological expectations of 'conical flow' [31].

[1]Notice, however, that the putative similarity in viscosity does not hold at *weak* coupling [11].

[2]The corresponding weakly-coupled calculation was carried out in [18].

An important measure of energy loss used in phenomenological models of medium-induced radiation (for reviews see [32]) is the jet-quenching parameter $\hat{q}$, defined as the average squared transverse momentum transferred to the quark by the medium, per unit distance travelled. The authors of [33] suggested that $\hat{q}$ could be identified with the logarithm of a certain lightlike Wilson loop,[3] and then used AdS/CFT to compute the latter for $\mathcal{N}=4$ SYM. The authors of [16] argued that a prediction for $\hat{q}$ in $\mathcal{N}=4$ SYM could be extracted, under certain assumptions, from their value for the drag force through use of the Langevin equation. Their result does not agree with that of [33], so there is some controversy on whether the lightlike Wilson loop employed in the latter work really computes the jet quenching parameter as defined in [32]. Be that as it may, a number of subsequent works have applied the prescription of [33] in more elaborate contexts [36, 37, 24, 38 – 40, 26], finding results whose qualitative form resembles that of corresponding drag forces [21]–[27], even if the detailed functional form of the two sets of quantities generally disagrees.

Several works have also considered the case where the plasma is probed not with a single quark but with a quark-antiquark pair. Such mesons were found to feel no drag force [41, 34, 35]: being color-neutral, they do not set up the long-range gluonic fields[4] that could transport energy away from them. This, however, is only true as long as the quark and antiquark are bound, which is of course made difficult by the screening effect of the plasma. The relevant q-$\bar{q}$ potential in the presence of a strongly-coupled $\mathcal{N}=4$ SYM plasma was computed some time ago in the case where the pair is static with respect to the plasma [43, 44], and recently in the more general case where the pair moves with an arbitrary velocity [35]. The upshot is that the quark and antiquark become unbound, and consequently experience a drag force, if their separation exceeds a certain screening length whose velocity- and temperature-dependence was obtained in [34, 35] (see also [41]). Various interesting extensions and refinements of these calculations have been performed in [45 – 48]. As first emphasized in [34], the results should have implications for charmonium suppression in the sQGP.

In this paper we take yet another step along this road by considering additional moving probes of the $\mathcal{N}=4$ SYM plasma, and using the AdS/CFT toolbox to determine their rates of energy loss. The probe of primary phenomenological interest is a gluon, because hard partons traversing the sQGP are expected to be gluons somewhat more often than they are quarks (in spite of which the hadrons with high transverse momentum detected after the collision come primarily from quarks [49]). One of our aims in this paper is therefore to compute the drag force experienced by a heavy gluon. A sketch of how one may go about doing this using a string and an antistring was given previously in [50] (a related discussion may be found in [48]).

We will model the gluon here as a pointlike external color source in the adjoint repre-

[3]Later work emphasized that this lightlike loop can be continuously connected with neighboring *timelike* loops only if the latter are taken to be traced by a source that is not completely pointlike [34, 35]. Alternatively, the loop of [33] may be thought of as a limit of *spacelike* loops traced by an ordinary pointlike source [35]. The physical significance of either of these statements remains to be understood; we will come back to the second statement towards the end of section 3.

[4]The color field profile set up by a meson was explicitly determined in [42] in the zero-temperature case.

sentation. From the theoretical perspective, it is also interesting to consider probes of the plasma that transform in other representations of the SU(N) gauge group. The technology for describing such sources in the dual AdS language has been developed only recently, in the context of Wilson loop computations.

The recipe for calculating Wilson loops in the fundamental representation has been known for a long time [51 – 53], and involves the identification of a string as the AdS counterpart of a fundamental color source. The case of Wilson loops in higher representations was discussed qualitatively already in [54] in terms of a collection of coincident strings, but only began to be understood more systematically after the beautiful work [55]. It was shown there that a D3-brane that carries electric flux[5] (i.e., a D3-F1 bound state) provides a more accurate description that correctly captures the effects of the interactions among the strings, which give rise to an infinite series of non-planar corrections in the gauge theory side [58]. This idea was elaborated on in interesting ways in the three simultaneous works [59 – 61]. The last paper, in particular, provided an elegant and complete dictionary for calculating Wilson loops in an arbitrary representation, using a collection of *either* D3-branes *or* D5-branes that carry electric flux. This prescription was derived explicitly in [61] for the special case of half-BPS Wilson loops in zero-temperature $\mathcal{N} = 4$ SYM, but given the motivation of [55] it should apply much more generally, as has indeed been assumed in subsequent works [62 – 67]. For our purposes, the bottom line is that the results of [61, 60] unambiguously identify D3-branes or D5-branes with appropriate electric fluxes as the AdS counterparts of the various external color sources that we wish to push through the SYM plasma.

To start out with, we will employ this identification to compute the dissipative force experienced by a collection of k quarks in the totally antisymmetric representation, which according to [60, 61] is dual to a single D5-brane with k units of electric flux. Arbitrary Wilson loops in this same representation were calculated recently in the nice paper [63], and the D5-brane we need to consider here is just a particular case of the ones studied there. We begin in section 2 by setting up the problem and working out the D5-brane embedding of interest. In the process, we discover that the constant-polar-angle D5-brane configurations (2.10)–(2.11) obtained in [68, 69], which are basic building blocks in the dictionary of [60, 61], are in fact incomplete: they are missing an 'end cap' that lies at the AdS boundary and whose presence is crucial to reproduce the correct energy and SU(4) charge of the dual k-quark system.[6] The complete D5-brane embeddings turn out to be a special case of the solutions obtained in the earlier work [70]; they can be described as the $S \to \infty$ limit of (2.12).

In section 3 we then extract from these D5-branes the value of the drag force exerted by the $\mathcal{N} = 4$ SYM plasma on their dual k-quark bound states. Our result is given in (3.2), which contains a non-trivial k-dependence displayed in figure 1. We verify that for $k = 1$ and large N our result matches the drag force experienced by the trailing string

[5] The relevance of D3-branes had been noted already in the original work [52], based on the Born-Infeld string construction [56].

[6] In the extremal case, it is only after including the contribution of this end cap that the energy of the system is proportional to its charge, resolving the puzzle encountered in [68].

of [16, 17], but (3.2) contains in addition an infinite series of $1/N$ corrections which in analogy with [55] should codify self-interactions of the string, arising from worldsheets with an arbitrary number of handles.

From the gauge theory viewpoint, a totally antisymmetric bound state of $(N-k)$ quarks should be equivalent to a totally antisymmetric bound state of k antiquarks, which by the symmetry between quarks and antiquarks leads to the prediction that the force experienced by a system of k quarks must be the same as the one felt by $N-k$ quarks, precisely as we find in (3.2). This implies in particular that the force vanishes for the case of the baryon, $k=N$, which seems natural given the no-drag result for the meson [41, 34, 35].

We also notice an intriguing connection between our drag force result (3.2) for a k-quark in $\mathcal{N}=4$ four-dimensional SYM and the tension $\sigma^{(k)}$ that governs the area law for spatial Wilson loops in the same theory (or, equivalently, the tension of a chromoelectric k-string in the *three*-dimensional non-supersymmetric *confining* theory associated with doubly-Wick-rotated D3-branes compactified on a small thermal circle [71]). The precise relation between the two quantities is given in (3.4), which appeared previously in the proposal of [23] relating parton energy loss to magnetic confinement. We observe that (3.4) also correctly reproduces some of the previously computed drag forces [22, 27], but not others [24 − 26], so as we discuss around (3.5), it would be interesting to establish whether some type of generalization holds in a more general context.

In section 4 we finally proceed to the case of a gluon, which the dictionary of [61] translates into a system of two D5-branes that respectively carry $k=N-1$ and $k=1$ units of electric flux. This system is described at tree-level by the non-Abelian action (4.3) plus an infinite series of corrections involving further commutators and derivatives of the fields. In the $N\to\infty$ limit the two D5-branes as expected decouple and their total energy loss rate is simply the sum of the rates of the individual branes, which is given by our previous result (3.2). The conclusion is that, at large N, the dissipative force experienced by a heavy gluon is as indicated in (4.5) double the size of the one felt by a heavy quark, precisely as group theory predicts on the gauge theory side. The computation of $1/N$ corrections to this result would require knowledge of the action for the two-D5-brane system at the level of the annulus and beyond.

All the color sources considered in this paper are for simplicity taken to be infinitely heavy, just like the external quark considered originally in [17]. It should however be possible to treat the case of a gluon/baryon/k-quark with finite mass by introducing D7-branes [72], in parallel with what was done for the single quark in [16]. Estimates of the thermal mass of the gluon in the real-world sQGP may be found for instance in [73].

2. D5-brane embedding

According to the prescription of [60, 61], in order to study a bound state of quarks in the totally antisymmetric representation of SU(N) moving through a thermal $\mathcal{N}=4$ SYM plasma, we must consider a D5-brane with electric flux that lives in the (Schwarzschild-

JHEP02(2007)084

AdS)$_5 \times \mathbf{S}^5$ geometry

$$ds^2 = \frac{1}{\sqrt{H}}\left(-hdt^2 + d\vec{x}^2\right) + \frac{\sqrt{H}}{h}dr^2 + R^2 d\Omega_5^2\,, \tag{2.1}$$
$$H = \frac{R^4}{r^4}\,, \qquad h = 1 - \frac{r_H^4}{r^4}\,, \qquad R^4 = 4\pi N g_s l_s^4\,,$$

with self-dual Ramond-Ramond field strength

$$G_{(5)} \equiv dC_{(4)} = 4R^4\left(\mathrm{vol}(\mathbf{S}^5)d\theta_1 \wedge \ldots \wedge d\theta_5 - \frac{r^3}{R^8}dt \wedge \ldots \wedge dx_3 \wedge dr\right)\,, \tag{2.2}$$

where $\mathrm{vol}(\mathbf{S}^5) \equiv \sin^4\theta_1 \mathrm{vol}(\mathbf{S}^4) \equiv \sin^4\theta_1 \sin^3\theta_2 \sin^2\theta_3 \sin\theta_4$. The D5-brane couples to these fields through the Born-Infeld plus Chern-Simons action

$$S_{D5} = T_{D5}\int d^6\sigma \left[-\sqrt{-\det\left(g_{ab} + 2\pi\alpha' F_{ab}\right)} + \left(2\pi\alpha' F_{(2)} \wedge c_{(4)}\right)_{0\ldots5}\right]\,, \tag{2.3}$$
$$g_{ab} \equiv \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}\,, \qquad c_{a_1\ldots a_4} \equiv \partial_{a_1}X^{\mu_1}\ldots\partial_{a_4}X^{\mu_4}C_{\mu_1\ldots\mu_4}\,.$$

We are instructed by [60, 61] to search for embeddings where the D5-brane extends radially in AdS$_5$, carries a radial electric field and wraps an $\mathbf{S}^4 \subset \mathbf{S}^5$, so it is convenient to choose the static gauge $\sigma^a = (t, r, \theta_2, \ldots, \theta_5)$, and to denote $\theta \equiv \theta_1$. Configurations of this type were studied previously in [74, 70, 68, 69]. As in [16, 17], we require a solution where the brane moves at constant speed along direction $x \equiv x^1$, described by a general stationary ansatz

$$X(t,r) = vt + \xi(r), \qquad \theta(r), \qquad X^2 = X^3 = 0\,. \tag{2.4}$$

Under these circumstances, the Chern-Simons term in (2.3) will involve only the $\theta_2 \ldots \theta_5$ component of the R-R gauge field. Integrating $\partial_{\theta_1}C_{\theta_2\ldots\theta_5} = G_{\theta_1\ldots\theta_5}$, we find it to be

$$C_{\theta_2\theta_3\theta_4\theta_5} = -R^4 D(\theta)\mathrm{vol}(\mathbf{S}^4)\,, \tag{2.5}$$

where as in [70] we have defined

$$D(\theta) \equiv -\frac{3}{2}\theta + \frac{3}{2}\sin\theta\cos\theta + \sin^3\theta\cos\theta\,. \tag{2.6}$$

Notice that the value of the integration constant in this expression figures in the generalized Wilson loop $\int_{\mathbf{S}^4} C_{(4)} \propto D(\theta)$, and consequently affects the physics. Just as in [68, 69], we have set it equal to zero to ensure that the solutions to follow have a consistent physical interpretation.[7]

Plugging (2.4) and (2.5) into the action (2.3), the Born-Infeld and Chern-Simons terms are both found to be proportional to $\mathrm{vol}(\mathbf{S}^4)$, whose integral yields the $\mathbf{S}^4$ volume $\Omega_4 = 8\pi^2/3$. Making use of $T_{D5} = 1/(2\pi)^5 g_s l_s^6$ and the definition of R in (2.1), we are then left with

$$S_{D5} = \frac{N}{3\pi^2\alpha'}\int dtdr\left[-\sin^4\theta\sqrt{-g^{(F1)} - (2\pi\alpha' F_{\mathrm{tr}})^2} - 2\pi\alpha' F_{\mathrm{tr}} D\right]\,, \tag{2.7}$$

[7] The only other consistent choice for this constant turns out to be $3\pi/2$, as in [63], and leads to equivalent physics.

where

$$g^{(F1)} \equiv (G_{tt} + v^2 G_{xx})(G_{rr} + \theta'^2 G_{\theta\theta}) + G_{tt} G_{xx} X'^2 \tag{2.8}$$

is the determinant of the metric that would have been induced on the worldsheet of a *string* embedded in the geometry (2.1) according to (2.4), and primes denote derivatives with respect to r.

The equations of motion for A_t and A_r imply that the momentum conjugate to A_r is a constant, which the flux quantization condition [75] requires to be an integer:

$$\Pi^t_{A_r} \equiv \frac{\partial \mathcal{L}_{D5}}{\partial(\partial_t A_r)} = \frac{2N}{3\pi} \left(\frac{E \sin^4 \theta}{\sqrt{-g^{(F1)} - E^2}} - D \right) = k \in \mathbf{Z}\,, \tag{2.9}$$

where we have set $E \equiv 2\pi\alpha' F_{\mathrm{tr}}$. As is familiar from the Born-Infeld string context [56], the integer k measures the fundamental string charge carried by the D5-brane: $\partial \mathcal{L}_{D5}/\partial B_{\mathrm{tr}} = k/2\pi\alpha'$. It is worth emphasizing, however, that according to (2.9) this charge receives a contribution not only from the electric field E that is relevant already in a trivial background, but also from the R-R four-form (2.5). For a given value of k, the equation of motion for θ allows a solution where the D5-brane sits at a specific constant polar angle

$$\theta = \Theta_k\,, \qquad \pi \frac{k}{N} = \Theta_k - \sin\Theta_k \cos\Theta_k\,, \tag{2.10}$$

as long as the electric field is given by

$$E = \pm \cos\Theta_k \sqrt{-g^{(F1)}}\,, \tag{2.11}$$

with the sign chosen to coincide with that of $\sin\Theta_k$.

Notice from (2.10) that $\Theta_{-k} = -\Theta_k$, which in (2.11) implies that $E(-k) = -E(k)$. This encodes the expected relation between the configurations with k strings and k anti-strings (i.e., strings with opposite orientation), which are respectively dual to k quarks and k antiquarks: the corresponding D5-branes have exactly the same embedding but opposite orientation, and carry electric fields which are equal in magnitude but opposite in sign. There is also a close connection between the systems with k and $N-k$ strings/quarks: it follows from (2.10) that $\Theta_{N-k} = \pi - \Theta_k$, meaning that the corresponding D5-branes wrap four-spheres located at the same angle Θ_k, but measured respectively from the north and south pole of the $\mathbf{S}^5$,[8] and (2.11) then implies that $E(N-k) = -E(k)$. The gauge theory interpretation of this symmetry will become clear below.

In the extremal case $r_H = 0$, Lorentz invariance of (2.1) along the boundary directions guarantees that the configuration (2.4) where the D5-brane moves along x with speed v is just a boosted version of the static solution, so we must have $\xi(r) = 0$ and therefore $X' = 0$, i.e., the brane remains upright. The authors of [60, 61] worked precisely in this case, and identified the constant-polar-angle solution (2.10)–(2.11), first obtained in [68], as the AdS counterpart of a bound state, in zero-temperature $\mathcal{N} = 4$ SYM, of k

[8] Choosing the integration constant in (2.6) as in the previous footnote would simply have exchanged the role of the north and south poles.

external quarks in the totally antisymmetric representation, moving at constant velocity v. Their identification was principally based on the fact that, just like a purely radial fundamental string, the above D5-brane embeddings preserve the same fermionic and bosonic symmetries as the required gauge theory sources: they are half-BPS and invariant under a $\mathrm{SO}(2,1) \times \mathrm{SO}(3) \simeq \mathrm{SU}(1,1) \times \mathrm{SU}(2)$ subgroup (generated by P_0, K_0, D and J_1, J_2, J_3) of the $\mathrm{SO}(4,2)$ conformal group, as well as a $\mathrm{SO}(5)$ subgroup of the $\mathrm{SO}(6) \simeq \mathrm{SU}(4)$ R-symmetry group— altogether, they are $Osp(4^*|4)$ invariant. A puzzling feature of these configurations, emphasized already in [68], is that their energy is not proportional to their charge. We will come back to this puzzle below.

Interestingly, in work prior to [68], a seemingly different family of D5-brane configurations that are also half-BPS was obtained and proposed to describe a k-quark bound state [70]. These are also solutions of the equations that follow from the extremal version of (2.7), but have a non-trivial profile $\theta(r)$, whose inverse was determined analytically in [70],

$$r(\theta) = \frac{S}{\sin\theta}\left[\frac{(\pi k/N - \theta + \sin\theta\cos\theta)}{\pi k/N}\right]^{1/3}, \tag{2.12}$$

with S an arbitrary constant, and carry a corresponding electric field as dictated by (2.9). These solutions cover the angular range $0 \leq \theta \leq \Theta_k$ (where the expression inside the brackets is positive) and have r decreasing monotonically from ∞ at $\theta = 0$ to 0 at $\theta = \Theta_k$. They describe a bundle of k fundamental strings that extend radially from the boundary at $r \to \infty$ to the horizon at $r = 0$, pointing always towards the north pole of $\mathbf{S}^5$, $\theta = 0$.

The embeddings (2.12) were shown to be BPS from the point of view of supersymmetry preservation in [74, 76], and from the perspective of energy minimization in [70, 76, 77]. They leave 16 supersymmetries unbroken, and have an energy equal to that of k fundamental strings, supporting their identification as threshold bound states of k quarks in $\mathcal{N} = 4$ SYM [70]. This holds for any value of the scale factor S, which is therefore a modulus in the space of equal-energy configurations. Through the standard UV-IR connection [78], on the gauge theory side $L \equiv R^2/S$ is a measure of the spatial extent of the k-quark bound state, as seen most explicitly in [42] for the case $k = N$. The existence of solutions with arbitrary size is of course a consequence of the conformal invariance of $\mathcal{N} = 4$ SYM.

Given that both (2.10)–(2.11) and the one-parameter family (2.12) are half-BPS D5-brane configurations claimed to be dual to k-quark bound states, it is natural to wonder whether they are related in some way. An important clue in this regard comes from the fact that, although the fermionic symmetries they preserve are the same, their bosonic symmetries do not quite coincide. For general scale factor S, the embedding (2.12) is, just like (2.10)–(2.11), invariant under $\mathrm{SO}(5) \subset \mathrm{SO}(6)$ and $\mathrm{SO}(3) \subset \mathrm{SO}(4,2)$, but not under $\mathrm{SO}(2,1) \subset \mathrm{SO}(4,2)$, because the dilatation generator D sends $(t, \vec{x}, r) \to (\lambda t, \lambda\vec{x}, r/\lambda)$ and consequently rescales the value of S.

The only exception to this is the case with $S \to \infty$ (corresponding to a gauge theory source with size $L \to 0$), which *is* invariant under dilatations, and, altogether, under precisely the *same* $Osp(4^*|4)$ as the constant-angle embedding found in [68]. The reason for this agreement is plain to see: in the $S \to \infty$ limit, the portion of (2.12) lying near

$r = 0$ and $\theta = \Theta_k$ is blown up to coincide *exactly* with (2.10)–(2.11)! The constant-angle D5 embedding obtained in [68] is in this way understood to be a particular member of the one-parameter family of configurations found earlier in [70]. Moreover, the solution of [68] is seen to be incomplete, because it is missing the portion of (2.12) that covers the angular range $0 \leq \theta < \Theta_k$, which is pushed by the $S \to \infty$ limit all the way out to the AdS boundary $r \to \infty$. To put it in more graphic language, each of the polar plots of (2.12) shown in figure 1 of [70] can be described as a small 'cone' joined onto a long 'tube', and the solution (2.10)–(2.11) is obtained, after an infinite rescaling, by keeping the now infinitely large cone but ignoring the tube, which now lies at the boundary and can be thought of as an end cap for the cone.

This last observation resolves the energy-charge puzzle that appeared in [68]: once the end cap is included, the total energy of the configuration *is* found to be correctly proportional to its total charge, as demonstrated in [70].[9] This clarification is particularly important in the case of the baryon, $k = N$, where we find $\Theta_N = \pi$ meaning that the cone (2.10)–(2.11) has collapsed to become a tensionless string, but the end cap now consists of a D5-brane fully wrapped around the $\mathbf{S}^5$ at $r \to \infty$ and carries an energy equal to that of N quarks.

The presence of the end cap also plays a role in explaining how the D5 embedding codifies the SU(4) quantum numbers of the quarks. Indeed, unlike the cone (2.10), which lies in the northern or southern hemisphere of $\mathbf{S}^5$ depending on whether $k < N/2$ or $k > N/2$, the end cap included in the full solution (2.12) always closes off at the *north* pole, correctly reflecting the fact that the D5-brane in question describes a bundle of strings (or antistrings) pointing towards $\theta = 0$, and is consequently dual to k quarks (or antiquarks) which source only the specific SYM scalar field that defines the polar direction of the internal space. These solutions were called 'lower tubes' in [70]. That same work constructed as well 'upper tube' solutions that always close off at the *south* pole and describe a bundle of antistrings (or strings) pointing towards $\theta = \pi$.[10] The embedding of this type that, based on energetics, corresponds to a bound state of k antistrings (or strings), is found to consist of a cone at $\theta = \Theta_{N-k}$ and an end cap that covers the angular range $\Theta_{N-k} < \theta \leq \pi$. By symmetry, the angle that this 'upper' cone makes with the south pole, $\pi - \Theta_{N-k}$, must coincide with the angle Θ_k that the 'lower' cone describing k antistrings (or strings) makes with the north pole. This explains the previously mentioned connection $\Theta_{N-k} = \pi - \Theta_k$, which we now recognize as the AdS counterpart of the gauge-theoretic statement that a totally-antisymmetric bound state of $N - k$ quarks with a given SU(4) charge is equivalent to a totally-antisymmetric bound state of k *anti*quarks with the *opposite* SU(4) charge.

In the last few paragraphs we have focused on the extremal background, which was the arena of [70, 68, 60, 61]. Since in this paper we wish to study the effects of a ther-

[9] Notice that the discussion in [70] of a 'missing charge' for the cases with $k \neq 0, N/2, N$ was misguided: the relevant charge is in all cases given by k as in (2.9), and *not* by the total R-R flux intercepted by the D5-brane.

[10] These are comparable to the solutions of [68] that result from the alternative choice of integration constant described in footnotes 7 and 8.

mal $\mathcal{N} = 4$ SYM plasma, we are mostly concerned with D5-brane embeddings in the *non*-extremal geometry (2.1) with $r_H > 0$. As we have seen above, in that case the constant-angle embedding (2.10)–(2.11) is still a solution, as was first proven in [63]. The configurations (2.12), on the other hand, do not admit a straightforward generalization to the non-extremal case [71, 79]. An important point is that the presence of the black hole in AdS breaks supersymmetry and conformal invariance, so S can no longer be a modulus, and we would *not* in fact expect to find a one-parameter family of solutions directly generalizing (2.12). But given that the cone (2.10) *is* a solution, it should be the case that the $S \to \infty$ limit of (2.12) in which it is contained is also a solution. To verify this one need only show that the portion of the D5-brane which is contained in the full embedding (2.12) and not in (2.10), i.e., the end cap, solves the non-extremal equation of motion, but this is evidently true because this portion lies entirely at $r \to \infty$, where the terms in the equation arising from the non-extremality are subleading. This is then another important property that distinguishes the $S \to \infty$ configurations from its $S < \infty$ siblings: among the k-quark bound states found in [70], it is the only one that admits a direct generalization away from the extremal case. Clearly it is this embedding alone that will provide us with the pointlike ($L = 0$) source that we wish to drag through the thermal plasma, and for this reason in the sections to follow we will concentrate exclusively on it.

3. Drag force on k-quarks and baryons

In the previous section we have learned that a bound state of k quarks in the totally antisymmetric representation moving through a thermal $\mathcal{N} = 4$ SYM plasma is described in dual language by a D5-brane on (Schwarzschild-AdS)$_5 \times \mathbf{S}^5$ whose $\theta(r)$ profile is given by the $S \to \infty$ limit of (2.12), which consists of two parts: the 'cone' (2.10)–(2.11), which extends from the horizon $r = r_H$ to the boundary $r \to \infty$, and an 'end cap' located at the boundary and covering the angular range $0 \leq \theta < \Theta_k$. As we have seen, the inclusion of this end cap is important to reproduce the correct energy and SU(4) charge of the system of k quarks, and in fact the *same* cone, with a $\Theta_k < \theta \leq \pi$ end cap, is dual to a bound state of $N - k$ antiquarks. We would now like to determine the force F_x with which this D5-brane must be pulled to maintain its velocity, which is mapped by AdS/CFT into the drag force experienced by the k-quark (or $(N-k)$-antiquark). As in [16, 17], this force will be non-zero only if the moving D5-brane leans back, i.e., if the function $\xi(r)$ describing the profile (2.4) of the D5-brane in the direction of motion is non-trivial. Since the end cap lies only at the boundary, for the purpose of determining this profile we need only concentrate on the conical portion of the solution.

For the cone (2.10)–(2.11), the equation of motion for $X(r,t)$ that follows from (2.7) is easily seen to be the same as the one that would be obtained from the Nambu-Goto Lagrangian $-\sqrt{-g^{(F1)}}$, which describes a *string* embedded in (Schwarzschild-AdS)$_5$ according to (2.4), with fixed $\theta_1, \ldots, \theta_5$. This is just a particular case of the result obtained in [63], where it was shown that, given *any* solution to the equations of motion for a string that reaches the boundary of an asymptotically AdS$_5$ spacetime M, one can obtain a one-parameter family of valid D5-brane embeddings on $M \times \mathbf{S}^5$ (where the sphere is threaded

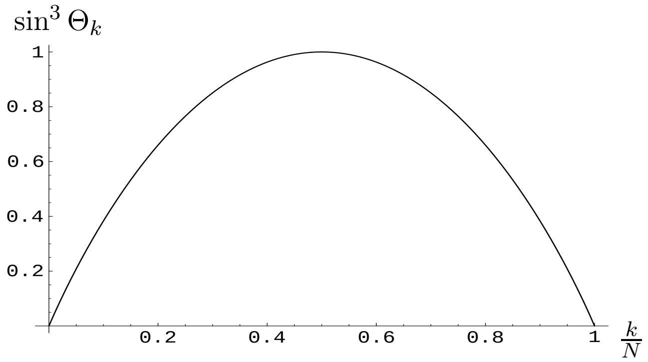

Figure 1: k-dependence of the drag force experienced by a k-quark.

by N units of R-R flux), simply by taking the D5 to carry k units of electric flux and wrap the $\mathbf{S}^4 \subset \mathbf{S}^5$ at polar angle $\theta = \Theta_k$. Based on this general result, the drag force we obtain below in the context of $\mathcal{N} = 4$ SYM should also be relevant for *any* thermal gauge theory with a dual description in terms of Type IIB string theory on $M \times \mathbf{S}^5$.

Because the reduced dynamics coincides with that of a string, the situation is exactly the same as the one examined in [16, 17]: for stationary configurations of the type (2.4), the equation of motion for $X(r,t)$ implies that the conjugate momentum density

$$\Pi_x^r \equiv \frac{\partial \mathcal{L}_{D5}}{\partial X'} = \frac{2N \sin^3\Theta_k}{3\pi} \frac{1}{2\pi\alpha'} \left(\frac{G_{tt} G_{xx} X'}{\sqrt{-g^{(F1)}}} \right) \tag{3.1}$$

is a constant, and there is a critical radius $r_v \equiv r_H/(1-v^2)^{1/4}$ below which the solution is found to be real only if the factor within parentheses takes the precise value $-(r_H/R)^2(v/\sqrt{1-v^2})$. This then fixes the value of the momentum density (3.1), which measures the rate at which x-momentum flows radially along the D5-brane, away from the boundary and towards the horizon. To keep it moving at constant velocity v along direction x, an external agent must supply to the D5-brane precisely this amount of linear momentum at the boundary, i.e., it must exert a force of magnitude (3.1).

According to the AdS/CFT dictionary established in [60, 61], the force exerted on the D5-brane translates into the drag force experienced by a system of k quarks (or $N-k$ antiquarks) in the totally antisymmetric representation that ploughs through a thermal $\mathcal{N} = 4$ SYM plasma. Through use of the relations $T = r_H/\pi R^2$ and $R^4 = g_{\mathrm{YM}}^2 N l_s^2$, this dissipative force may be written in the final form

$$F_x^{(k)} \equiv \frac{dp_x}{dt} = \frac{2N}{3\pi} \sin^3\Theta_k \left(-\frac{\pi}{2} \sqrt{g_{\mathrm{YM}}^2 N} T^2 \frac{v}{\sqrt{1-v^2}} \right), \tag{3.2}$$

which is our main result in this section. The momentum flow (3.2) from the moving k-quark to the plasma implies a corresponding energy loss rate $dE/dt = v dE/dx = v F_x^{(k)}$ (which can also be inferred directly from the D5-brane momentum density Π_t^r).

It is interesting that the k-dependence is contained entirely in the $\sin^3\Theta_k$ prefactor, whose behavior is shown in figure 1, and is such that $F_x^{(N-k)} = F_x^{(k)}$, exhibiting the expected symmetry between quarks and antiquarks. This implies in particular that the bound state of N quarks, the baryon, must feel the same force as a system with no quarks at all, which is to say that $F_x^{(N)} = 0$. We thus learn that, at this level of approximation, the baryon can traverse the plasma without suffering any energy loss. This is the same conclusion as was reached for the meson in [41, 34, 35]. The reason in both cases is the same: neither of these color-neutral sources are able to set up the long-range gluonic fields[11] that are responsible for carrying energy away from the k-quarks with $0 < k < N$, as has been studied in detail in [28, 30]. In fact, for $k = N$ the angle (2.10) becomes $\theta = \pi$, meaning that the 'conical' part of the D5-brane has closed off and become tensionless. Since it is this portion that codifies the gluonic fields set up by the N-quark system, we conclude that our pointlike baryon produces no field at all. It is in effect described solely by the D5 'end cap' at the AdS boundary, which in this case is completely wrapped around $\mathbf{S}^5$, and is able to move unimpeded because, lying at $r \to \infty$, it is unaffected by the presence of the horizon.

It is instructive to consider the case of a single quark, $k = 1$ (or a single antiquark, $k = N-1$), where the D5-brane lies at a polar angle Θ_1 (or $\pi - \Theta_1$) which according to (2.10) is given by $\pi/N = \Theta_1 - \sin\Theta_1 \cos\Theta_1$. For $N \gg 1$, this implies that $\sin^3\Theta_1 \simeq \Theta_1^3 \simeq 3\pi/2N$, so as required the drag force $F_x^{(1)}|_{N\to\infty}$ agrees with the one obtained in [16, 17] (the factor inside the parentheses in (3.2)), whose computation was carried out at string tree-level, and therefore applies in the large N limit. Based on the results of [55], it is tempting to speculate that the finite N corrections to this result that can be deduced from the D5-brane drag force (3.2) encode higher-order corrections that arise from self-interactions of the string. More generally, if we hold k fixed in the large N limit, it follows from (3.2) that

$$F_x^{(k)} = k\left(1 - \frac{3}{10}\left(\frac{3\pi k}{2N}\right)^{2/3} + \ldots\right) F_x^{(1)}|_{N\to\infty}\,, \tag{3.3}$$

and we would expect the second and higher terms to capture the effect of the interactions of the k quarks among themselves. Notice that the first correction is negative, meaning that, at finite g_{YM}, the k-quark loses energy at a somewhat smaller rate than k unbound quarks. It should of course be borne in mind that, since unlike [55] we are not working in a supersymmetric context, there might be additional finite-N contributions arising from string-loop corrections to the background (2.1).

The $\sin^3\Theta_k$ prefactor seen in our drag force result (3.2) also appeared in the Wilson loop calculation of [63]. This is evidently not surprising from the string theory perspective, because both calculations use the same D5-brane embedding, the sole difference being that the value of the Wilson loop receives a contribution not only from the action (2.7) but also from various boundary terms that are responsible for yielding a finite answer [53, 55, 64]. The agreement is even more unsurprising from the gauge theory viewpoint, because both quantities refer to the same type of external source in the same thermal gauge theory.

What we do find interesting is that the very same factor $\sin^3\Theta_k$ showed up as well in [71], which dealt with a problem that from the field theory perspective would appear to be completely different: the computation of flux tube tensions $\sigma^{(k)}$ in the confining three-dimensional gauge theory that is obtained by dimensionally reducing Wick-rotated

[11]The chromoelectric field produced by the baryon has been determined explicitly in [42] at zero temperature.

JHEP02(2007)084

D3-branes on a supersymmetry-violating circle (in parallel with the D4-brane discussion of [80]). This problem was studied in [71] by splitting a baryon into two groups of k and $N-k$ quarks (or equivalently, k quarks and k antiquarks), thereby producing a long chromoelectric flux tube, a QCD k-string, in between. The dual description for this split baryon involves a D5-brane (as inferred from the picture developed for $\mathcal{N}=4$ SYM in [81, 74, 70]), and the portion of this brane that is dual to the flux tube turns out to lie precisely at the polar angle (2.10), which explains, from the string theory side, the agreement with our result.

The precise relation between the quantities computed here and in [71] is

$$F_x^{(k)} = -\sigma^{(k)} \frac{v}{\sqrt{1-v^2}}\,, \tag{3.4}$$

where it is seen that the force and the tension have identical dependence on k, N, the 't Hooft coupling and the temperature. Using a fundamental string instead of a D5-brane, the authors of [23] found this same relation to hold in the particular case $k=1$, $N\to\infty$. They interpreted $\sigma^{(k)}$ directly in the four-dimensional SYM theory as the tension controlling the area law for *spatial* Wilson loops [80], and used the fact that such loops codify the magnetic interaction between two current wires to argue that the agreement (3.4) reflects the magnetic origin of the drag force. We will return to this point below.

The computation of the k-string tension in d-dimensions is an important problem in the study of confining gauge theories (see, e.g., [82] for recent reviews, and the works [83] for related AdS/CFT calculations), and it would be worth exploring whether the connection obtained here and in [23] with a drag force calculation in $d+1$ dimensions holds more generally. For instance, the simple $(k/N)(1-k/N)$ scaling of the flux tube tension found in [71] for the four-dimensional Yang-Mills theory obtained from D4-branes [80] suggests an identical scaling for the rate of parton energy loss in the corresponding five-dimensional thermal plasma. Notice that the former quantity is static, whereas the latter is dynamical, so if one could establish a relation of the type (3.4) in a more general setting, it would in particular become possible to access information on energy loss from flux tube calculations on the lattice.

A formula like (3.4) would also greatly facilitate the AdS/CFT computation of parton energy loss in other theories, because on the geometry side the value of $\sigma^{(k)}$ is simply given by the tension of a string/brane that is localized in time and lies at the stationary limit surface $r=r_s$ where $G_{tt}=0$. In the case of a source in the fundamental representation, which is modelled with a fundamental string, this amounts to $\sigma^{(1)} = G_{xx}(r_s)/2\pi\alpha'$ (with $G_{\mu\nu}$ the string frame metric). We have checked that when inserted in (3.4), this correctly reproduces the drag force computed in some cases [22, 27], but not in others [24–26], which generally involve non-trivial background fields other than the metric. In particular, the v-dependence of the actual drag force can be much more complicated than $v/\sqrt{1-v^2}$, so the desired generalization of (3.4), if it exists at all, should involve a prescription for correctly determining this dependence.

Given that in the drag force calculations a central role is played not by the stationary limit radius r_s but by the velocity-dependent radius r_v, it is natural to expect a relation between F_x and the tension of a spacelike string (or brane) localized at the latter radius. And indeed, working on an arbitrary background as in [21, 24] one can prove that in all cases

$$F_x = -\tilde{\sigma}\frac{v}{\sqrt{1+v^2}}\,, \tag{3.5}$$

where $\tilde{\sigma} = \sqrt{1+v^2}G_{xx}(r_v)/2\pi\alpha'$ is the tension of a string worldsheet extended along $\tilde{x}, x_2$ and localized at $\tilde{t} = $ const., $r = r_v$, with $(\tilde{t}, \tilde{x})$ the coordinates in the q-$\bar{q}$ rest frame and r_v the radius where $G_{\tilde{t}\tilde{t}} = 0$. This last condition shows that r_v marks the location of the stationary limit surface of the black brane *as seen by an observer at rest in the* $(\tilde{t}, \tilde{x})$ *frame* (and so in particular agrees with r_s when $v = 0$). All of this strongly suggests that (3.5) is the desired generalization of the force-tension relation (3.4). But for this to be useful, a simple gauge-theoretic interpretation should be found for the tension $\tilde{\sigma}$, which has thus far been defined only on the AdS side. The natural guess that it controls the area law for a spatial Wilson loop extended along $\tilde{x}, x_2$ turns out to be incorrect, because the $\cap$-shaped string that would be employed on the AdS side to compute such a loop descends beyond r_v and all the way down to r_s as the loop becomes large. This casts some doubt on the proposal of [23], because (3.4) appears to work only in those cases where the background is such that σ happens to be proportional to $\tilde{\sigma}$.

In any event, it should be noted that the observation made in [23] and the present paper that the information on the rate of parton energy loss can (at least in some cases) be extracted from a *spatial* Wilson loop is reminiscent of the proposal of [33] for computing the jet-quenching parameter using a *lightlike* Wilson loop, and in fact lends credence to the idea that the latter is to be regarded as a limit of a *spacelike* loop traced by a pointlike source whose velocity approaches the speed of light *from above* [35]. Notice, however, that the loops discussed here and in [33] explore different regimes: whereas the magnetic area law ($E \propto L$) is found to hold for wide loops ($LT \gg 1$), the 'dipole approximation' ($E \propto L^2$) that inspired [33] is valid only for narrow loops ($LT \ll 1$). It might prove worthwhile to investigate in detail the possible extrapolation between these two regimes, hopefully shedding light on the meaning of (3.5) in the process.

It would also be interesting to extend our drag force calculation to the case of external sources in representations other than the totally antisymmetric one. The simplest example would be a system of k quarks in the totally symmetric representation, which according to the dictionary of [61] is dual to a D3-brane that carries k units of fundamental string charge. The equations that determine the relevant D3-brane embedding are much more complicated than the ones for the D5-brane, so regrettably we must leave their analysis to future work.

4. Drag force on a heavy gluon

According to [61], in the same way we represent k quarks in the totally antisymmetric representation as a D5-brane with k units of fundamental string charge, we can think of a gluon (a source in the adjoint representation) as two parallel D5-branes, carrying $k = 1$ and $k = N - 1$ units of fundamental string charge respectively. The dynamics of this system is described by some U(2) action whose full form is not yet known. To date, the most complete proposal is that of [84] (see also [85]),

$$S_{Dp} = -T_p \int d^{p+1}\sigma \,\mathrm{Tr}\left(e^{-\phi}\sqrt{-\det\left(P\left[E_{\mu\nu} + E_{\mu i}(Q^{-1}-\delta)^{ij}E_{j\nu}\right] + 2\pi\alpha' F_{\mu\nu}\right)\det(Q^i_j)}\right)$$
$$+T_p \int \mathrm{Tr}\left(P\left[e^{i2\pi\alpha' \mathrm{i}_\Phi \mathrm{i}_\Phi}(\textstyle\sum C^{(n)}e^B)\right]e^{2\pi\alpha' F}\right) \tag{4.1}$$

where

$$E_{\mu\nu} = G_{\mu\nu} + B_{\mu\nu}, \qquad Q^i_j = \delta^i_j + i2\pi\alpha' \left[\Phi^i, \Phi^k\right] E_{kj}, \tag{4.2}$$

$P[E]$ is the general formula for the non-Abelian pullback and i_Φ denotes the interior product by Φ_i regarded as a vector in the transverse space. The gauge field is now non-Abelian and the scalars Φ^i belong to the adjoint U(2) representation. As first noticed by [86, 87], all derivatives in the action are replaced by covariant derivatives. Furthermore, all the bulk fields are functionals of the non-Abelian scalars. As in the Abelian case, these fields are interpreted in terms of a Taylor expansion, however, the transverse displacements are now matrix-valued, so the action would be given by a non-Abelian Taylor expansion [84].

Finally, the gauge trace in (4.1) is meant to be implemented as was first proposed in [86], taking the totally symmetrized product of all non-Abelian expressions of the form $F_{\mu\nu}$, $D_\mu\Phi^i$ and $\left[\Phi^i, \Phi^j\right]$. This prescription correctly yields the F^2 and F^4 interactions, but seems to require modifications at order F^6 and higher [88]. This shortcoming is related to the fact that the prescription of dropping all interactions involving derivatives of the field strength, as one does in the Abelian Born-Infeld action, is ambiguous in the non-Abelian theory because of the commutators that involve the gauge fields.

In our particular case, where $\phi = 0$, $B_{\mu\nu} = 0$ and the spacetime metric (2.1) is diagonal, (4.1) reduces to:

$$S_{D5} = -T_{D5}\int d^6\sigma\,\mathrm{Tr}\left[\sqrt{-\det\left(P[G_{\mu\nu}] + 2\pi\alpha' F_{\mu\nu}\right)} - \left(2\pi\alpha' F_{(2)} \wedge P[C_{(4)}]\right)_{0\cdots 5}\right] \tag{4.3}$$

with

$$P[G_{\mu\nu}] = G_{\mu\nu} + G_{ij}D_\mu X^i D_\nu X^j \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i\left[A_\mu, A_\nu\right] \tag{4.4}$$

Upon expanding both terms in (4.3) to get a polynomial expression in the fields, it is easy to show that, after taking the trace, all the off-diagonal elements (12 or 21) of the fields always appear in pairs. It is therefore consistent with the equations of motion to set all these elements equal to zero. Furthermore, all interactions between diagonal terms of type 11 and 22 involve also off-diagonal elements, and consequently setting the latter to zero turns off these interactions. So, at this order, the two D5-branes do not see one another, and we can work directly with two copies of the Abelian equations of motion.

It is important to notice that the above result applies in fact not only for the specific action (4.1), but also for whatever action codifies the complete tree-level D-brane interactions. The reason is that this action arises from disk diagrams, where the presence of a single boundary (responsible for the appearance of a single trace) forces each 12 vertex to be paired with a 21 vertex, and allows insertion of 11 and 22 vertices in a single diagram only in conjunction with at least one of these off-diagonal pairs.

JHEP02(2007)084

At this point, we can use our results from section 3. We have two decoupled D5-branes, one with $k = 1$ and the other with $k = N - 1$ units of electric flux. Using equation (3.2) we then get the net drag force on the gluon,

$$F(\text{gluon}) = F_x^{(1)} + F_x^{(N-1)} = 2F_x^{(1)} = 2F(\text{quark}) \; . \tag{4.5}$$

This result is in agreement with what is expected from the gauge theory side. The basic point is that, just from the Feynman rules for the three-point QCD vertices, one finds that the probability for a parton to radiate a gluon is proportional to the quadratic Casimir in the relevant representation of the SU(N) group, N if the parton is a gluon and $(N^2-1)/2N$ if it is a quark. The same group-theory factor appears in the more complicated calculations of parton energy loss [32], and for this reason the relative factor between the gluon and quark energy loss rate is expected to be $2N^2/(N^2-1)$ [49, 89], at least in the perturbative regime. In the large N limit, this reduces to a factor of 2, just as we have found here. In the real-world $N = 3$ case, the factor is not quite 2 but 2.25.

Our result should receive $1/N$ corrections, which must come from calculations at higher order in string loops. Starting at the level of the annulus, the presence of multiple boundaries (which results in multiple traces) allows simultaneous insertions of 11 and 22 vertices without the appearance of off-diagonal modes (which still come in pairs, and so can consistently be set to zero), meaning that we can no longer treat each D5-brane independently. These diagrams are suppressed by powers of $g_s = g_{\rm YM}^2/4\pi$, which for fixed $g_{\rm YM}^2 N$ is equivalent to powers of $1/N$. It would be interesting to work out these corrections, but it would require explicit knowledge of the higher-order form of the non-Abelian action.

Acknowledgments

We thank Alejandro Ayala, Eleazar Cuautle, Antonio García, Martín Kruczenski and Guy Paic for very valuable discussions and for pointing out several useful references. We are also grateful to Antonio García and Damián Hernández for comments on the manuscript. This work was partially supported by Mexico's National Council of Science and Technology (CONACyT) grants CONACyT 40754-F and CONACyT SEP-2004-C01-47211, as well as by DGAPA-UNAM grant IN104503-3.

References

[1] J.M. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [*Int. J. Theor. Phys.* **38** (1999) 1113] [hep-th/9711200].

[2] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett.* **B 428** (1998) 105 [hep-th/9802109];
E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [hep-th/9802150];

[3] O. Aharony, S.S. Gubser, J.M. Maldacena, H. Ooguri and Y. Oz, *Large-N field theories, string theory and gravity*, *Phys. Rept.* **323** (2000) 183 [hep-th/9905111].

JHEP02(2007)084

[4] PHENIX collaboration, K. Adcox et al., *Formation of dense partonic matter in relativistic nucleus nucleus collisions at RHIC: experimental evaluation by the PHENIX collaboration*, *Nucl. Phys.* **A 757** (2005) 184 [nucl-ex/0410003];
BRAHMS collaboration, I. Arsene et al., *Quark gluon plasma and color glass condensate at RHIC? the perspective from the BRAHMS experiment*, *Nucl. Phys.* **A 757** (2005) 1 [nucl-ex/0410020];
B.B. Back et al., *The phobos perspective on discoveries at RHIC*, *Nucl. Phys.* **A 757** (2005) 28 [nucl-ex/0410022];
STAR collaboration, J. Adams et al., *Experimental and theoretical challenges in the search for the quark gluon plasma: the star collaboration's critical assessment of the evidence from RHIC collisions*, *Nucl. Phys.* **A 757** (2005) 102 [nucl-ex/0501009].

[5] ALICE collaboration, F. Carminati et al., *ALICE: physics performance report, volume I*, *J. Phys.* **G 30** (2004) 1517;
ALICE collaboration, A. Morsch*Jet quenching studies with the ALICE detector*, *Czech. J. Phys.* **55** (2005) B333;
L. I. Sarycheva, *Heavy ion collisions at CMS and quark gluon plasma*, *Czech. J. Phys.* **52** (2002) C171.

[6] E.V. Shuryak, *Strongly coupled quark-gluon plasma: the status report*, hep-ph/0608177;
B. Muller and J.L. Nagle, *Results from the Relativistic Heavy Ion Collider*, nucl-th/0602029;
M.J. Tannenbaum, *Recent results in Relativistic Heavy Ion Collisions: from 'a new state of matter' to 'the perfect fluid'*, *Rept. Prog. Phys.* **69** (2006) 2005–2060 [nucl-ex/0603003];
M. Gyulassy and L. McLerran, *New forms of QCD matter discovered at RHIC*, *Nucl. Phys.* **A 750** (2005) 30 [nucl-th/0405013];
D.H. Rischke, *The quark-gluon plasma in equilibrium*, *Prog. Part. Nucl. Phys.* **52** (2004) 197 [nucl-th/0305030].

[7] S.S. Gubser, I.R. Klebanov and A.W. Peet, *Entropy and temperature of black* 3*-branes*, *Phys. Rev.* **D 54** (1996) 3915 [hep-th/9602135].

[8] F. Karsch, *Lattice QCD at high temperature and density*, *Lect. Notes Phys.* **583** (2002) 209–249 [hep-lat/0106019];
F. Karsch, E. Laermann and A. Peikert, *The pressure in* 2, $2+1$ *and* 3 *flavour QCD*, *Phys. Lett.* **B 478** (2000) 447 [hep-lat/0002003].

[9] G. Policastro, D.T. Son and A.O. Starinets, *The shear viscosity of strongly coupled* $N=4$ *supersymmetric Yang-Mills plasma*, *Phys. Rev. Lett.* **87** (2001) 081601 [hep-th/0104066].

[10] D. Teaney, *Effect of shear viscosity on spectra, elliptic flow and Hanbury Brown-Twiss radii*, *Phys. Rev.* **D 68** (2003) 034913 [nucl-th/0301099].

[11] S.C. Huot, S. Jeon and G.D. Moore, *Shear viscosity in weakly coupled* $N=4$ *super-Yang-Mills theory compared to QCD*, hep-ph/0608062.

[12] H. Nastase, *The RHIC fireball as a dual black hole*, hep-th/0501068;
H. Nastase, *More on the RHIC fireball and dual black holes*, hep-th/0603176.

[13] E. Shuryak, S.-J. Sin and I. Zahed, *A gravity dual of RHIC collisions*, hep-th/0511199;
S. Nakamura and S.-J. Sin, *A holographic dual of hydrodynamics*, *JHEP* **09** (2006) 020 [hep-th/0607123];
S.-J. Sin, S. Nakamura and S.P. Kim, *Elliptic flow, Kasner universe and holographic dual of RHIC fireball*, *JHEP* **12** (2006) 075 [hep-th/0610113].

[14] R.A. Janik and R. Peschanski, *Asymptotic perfect fluid dynamics as a consequence of AdS/CFT*, *Phys. Rev.* **D 73** (2006) 045013 [hep-th/0512162].

[15] J.J. Friess, S.S. Gubser, G. Michalogiorgakis and S.S. Pufu, *Expanding plasmas and quasinormal modes of anti-de Sitter black holes*, hep-th/0611005.

[16] C.P. Herzog, A. Karch, P. Kovtun, C. Kozcaz and L.G. Yaffe, *Energy loss of a heavy quark moving through $N = 4$ supersymmetric Yang-Mills plasma*, *JHEP* **07** (2006) 013 [hep-th/0605158].

[17] S.S. Gubser, *Drag force in AdS/CFT*, *Phys. Rev.* **D 74** (2006) 126005 [hep-th/0605182].

[18] P.M. Chesler and A. Vuorinen, *Heavy flavor diffusion in weakly coupled $N = 4$ super-Yang-Mills theory*, *JHEP* **11** (2006) 037 [hep-ph/0607148].

[19] J. Casalderrey-Solana and D. Teaney, *Heavy quark diffusion in strongly coupled $N = 4$ Yang-Mills*, *Phys. Rev.* **D 74** (2006) 085012 [hep-ph/0605199].

[20] S.-J. Sin and I. Zahed, *Holography of radiation and jet quenching*, *Phys. Lett.* **B 608** (2005) 265 [hep-th/0407215].

[21] C.P. Herzog, *Energy loss of heavy quarks from asymptotically AdS geometries*, *JHEP* **09** (2006) 032 [hep-th/0605191].

[22] E. Caceres and A. Guijosa, *Drag force in charged $N = 4$ SYM plasma*, *JHEP* **11** (2006) 077 [hep-th/0605235].

[23] S.-J. Sin and I. Zahed, *Ampere's law and energy loss in AdS/CFT duality*, hep-ph/0606049.

[24] E. Caceres and A. Guijosa, *On drag forces and jet quenching in strongly coupled plasmas*, *JHEP* **12** (2006) 068 [hep-th/0606134].

[25] T. Matsuo, D. Tomino and W.-Y. Wen, *Drag force in SYM plasma with B field from AdS/CFT*, *JHEP* **10** (2006) 055 [hep-th/0607178].

[26] E. Nakano, S. Teraguchi and W.-Y. Wen, *Drag force, jet quenching and AdS/QCD*, hep-ph/0608274.

[27] P. Talavera, *Drag force in a string model dual to large-N QCD*, *JHEP* **01** (2007) 086 [hep-th/0610179].

[28] J.J. Friess, S.S. Gubser and G. Michalogiorgakis, *Dissipation from a heavy quark moving through $N = 4$ super-Yang-Mills plasma*, *JHEP* **09** (2006) 072 [hep-th/0605292].

[29] Y.-h. Gao, W.-s. Xu and D.-f. Zeng, *Wake of color fileds in charged $N = 4$ SYM plasmas*, hep-th/0606266.

[30] J.J. Friess, S.S. Gubser, G. Michalogiorgakis and S.S. Pufu, *The stress tensor of a quark moving through $N = 4$ thermal plasma*, hep-th/0607022.

[31] E. Shuryak, *The conical flow from quenched jets in sqgp*, *Nucl. Phys.* **A 783** (2007) 31 [nucl-th/0609013].

[32] R. Baier, D. Schiff and B.G. Zakharov, *Energy loss in perturbative QCD*, *Ann. Rev. Nucl. Part. Sci.* **50** (2000) 37–69 [hep-ph/0002198];
R. Baier, *Jet quenching*, *Nucl. Phys.* **A 715** (2003) 209 [hep-ph/0209038];
M. Gyulassy, I. Vitev, X.-N. Wang and B.-W. Zhang, *Jet quenching and radiative energy loss in dense nuclear matter*, nucl-th/0302077;
A. Kovner and U.A. Wiedemann, *Gluon radiation and parton energy loss*, hep-ph/0304151.

[33] H. Liu, K. Rajagopal and U.A. Wiedemann, *Calculating the jet quenching parameter from AdS/CFT*, *Phys. Rev. Lett.* **97** (2006) 182301 [hep-ph/0605178].

[34] H. Liu, K. Rajagopal and U.A. Wiedemann, *An AdS/CFT calculation of screening in a hot wind*, hep-ph/0607062.

[35] M. Chernicoff, J.A. Garcia and A. Guijosa, *The energy of a moving quark-antiquark pair in an $N = 4$ SYM plasma*, *JHEP* **09** (2006) 068 [hep-th/0607089].

[36] A. Buchel, *On jet quenching parameters in strongly coupled non-conformal gauge theories*, *Phys. Rev.* **D 74** (2006) 046006 [hep-th/0605178].

[37] J.F. Vazquez-Poritz, *Enhancing the jet quenching parameter from marginal deformations*, hep-th/0605296.

[38] F.-L. Lin and T. Matsuo, *Jet quenching parameter in medium with chemical potential from AdS/CFT*, *Phys. Lett.* **B 641** (2006) 45 [hep-th/0606136].

[39] S.D. Avramis and K. Sfetsos, *Supergravity and the jet quenching parameter in the presence of R-charge densities*, *JHEP* **01** (2007) 065 [hep-th/0606190].

[40] N. Armesto, J.D. Edelstein and J. Mas, *Jet quenching at finite 't Hooft coupling and chemical potential from AdS/CFT*, *JHEP* **09** (2006) 039 [hep-ph/0606245].

[41] K. Peeters, J. Sonnenschein and M. Zamaklar, *Holographic melting and related properties of mesons in a quark gluon plasma*, *Phys. Rev.* **D 74** (2006) 106008 [hep-th/0606195].

[42] C.G. Callan Jr. and A. Guijosa, *Undulating strings and gauge theory waves*, *Nucl. Phys.* **B 565** (2000) 157 [hep-th/9906153].

[43] S.-J. Rey, S. Theisen and J.-T. Yee, *Wilson-Polyakov loop at finite temperature in large-N gauge theory and anti-de Sitter supergravity*, *Nucl. Phys.* **B 527** (1998) 171 [hep-th/9803135].

[44] A. Brandhuber, N. Itzhaki, J. Sonnenschein and S. Yankielowicz, *Wilson loops in the large-N limit at finite temperature*, *Phys. Lett.* **B 434** (1998) 36 [hep-th/9803137].

[45] E. Caceres, M. Natsuume and T. Okamura, *Screening length in plasma winds*, *JHEP* **10** (2006) 011 [hep-th/0607233].

[46] P.C. Argyres, M. Edalati and J.F. Vazquez-Poritz, *No-drag string configurations for steadily moving quark-antiquark pairs in a thermal bath*, *JHEP* **01** (2007) 105 [hep-th/0608118].

[47] S.D. Avramis, K. Sfetsos and D. Zoakos, *On the velocity and chemical-potential dependence of the heavy-quark interaction in $N = 4$ sym plasmas*, *Phys. Rev.* **D 75** (2007) 025009 [hep-th/0609079].

[48] J.J. Friess, S.S. Gubser, G. Michalogiorgakis and S.S. Pufu, *Stability of strings binding heavy-quark mesons*, hep-th/0609137.

[49] A. Dainese, C. Loizides and G. Paic, *Leading-particle suppression in high energy nucleus nucleus collisions*, *Eur. Phys. J.* **C 38** (2005) 461 [hep-ph/0406201].

[50] S.S. Gubser, *Relativistic heavy ion collisions and string theory*, lectures at Prospects in Theoretical Physics, 2006, available online.

[51] J.M. Maldacena, *Wilson loops in large-N field theories*, *Phys. Rev. Lett.* **80** (1998) 4859 [hep-th/9803002].

JHEP02(2007)084

[52] S.-J. Rey and J.-T. Yee, *Macroscopic strings as heavy quarks in large-N gauge theory and anti-de Sitter supergravity*, *Eur. Phys. J.* **C 22** (2001) 379 [hep-th/9803001].

[53] N. Drukker, D.J. Gross and H. Ooguri, *Wilson loops and minimal surfaces*, *Phys. Rev.* **D 60** (1999) 125006 [hep-th/9904191].

[54] D.J. Gross and H. Ooguri, *Aspects of large-N gauge theory dynamics as seen by string theory*, *Phys. Rev.* **D 58** (1998) 106002 [hep-th/9805129].

[55] N. Drukker and B. Fiol, *All-genus calculation of Wilson loops using D-branes*, *JHEP* **02** (2005) 010 [hep-th/0501109].

[56] C.G. Callan Jr. and J.M. Maldacena, *Brane dynamics from the Born-Infeld action*, *Nucl. Phys.* **B 513** (1998) 198 [hep-th/9708147];

[57] G.W. Gibbons, *Born-Infeld particles and dirichlet p-branes*, *Nucl. Phys.* **B 514** (1998) 603 [hep-th/9709027];
P.S. Howe, N.D. Lambert and P.C. West, *The self-dual string soliton*, *Nucl. Phys.* **B 515** (1998) 203 [hep-th/9709014].

[58] N. Drukker and D.J. Gross, *An exact prediction of $N = 4$ SUSYM theory for string theory*, *J. Math. Phys.* **42** (2001) 2896 [hep-th/0010274].

[59] S.A. Hartnoll and S. Prem Kumar, *Multiply wound Polyakov loops at strong coupling*, *Phys. Rev.* **D 74** (2006) 026001 [hep-th/0603190].

[60] S. Yamaguchi, *Wilson loops of anti-symmetric representation and D5-branes*, *JHEP* **05** (2006) 037 [hep-th/0603208].

[61] J. Gomis and F. Passerini, *Holographic Wilson loops*, *JHEP* **08** (2006) 074 [hep-th/0604007].

[62] D. Rodriguez-Gomez, *Computing Wilson lines with dielectric branes*, *Nucl. Phys.* **B 752** (2006) 316 [hep-th/0604031].

[63] S.A. Hartnoll, *Two universal results for Wilson loops at strong coupling*, *Phys. Rev.* **D 74** (2006) 066006 [hep-th/0606178].

[64] S.A. Hartnoll and S.P. Kumar, *Higher rank Wilson loops from a matrix model*, *JHEP* **08** (2006) 026 [hep-th/0605027].

[65] B. Chen and W. He, *On 1/2-BPS Wilson-'t Hooft loops*, *Phys. Rev.* **D 74** (2006) 126008 [hep-th/0607024].

[66] S. Giombi, R. Ricci and D. Trancanelli, *Operator product expansion of higher rank Wilson loops from D-branes and matrix models*, *JHEP* **10** (2006) 045 [hep-th/0608077].

[67] T.-S. Tai and S. Yamaguchi, *Correlator of fundamental and anti-symmetric Wilson loops in AdS/CFT correspondence*, *JHEP* **02** (2007) 035 [hep-th/0610275].

[68] J. Pawelczyk and S.-J. Rey, *Ramond-Ramond flux stabilization of D-branes*, *Phys. Lett.* **B 493** (2000) 395 [hep-th/0007154].

[69] J.M. Camino, A. Paredes and A.V. Ramallo, *Stable wrapped branes*, *JHEP* **05** (2001) 011 [hep-th/0104082].

[70] C.G. Callan Jr., A. Guijosa and K.G. Savvidy, *Baryons and string creation from the fivebrane worldvolume action*, *Nucl. Phys.* **B 547** (1999) 127 [hep-th/9810092].

JHEP02(2007)084

[71] C.G. Callan Jr., A. Guijosa, K.G. Savvidy and O. Tafjord, *Baryons and flux tubes in confining gauge theories from brane actions*, *Nucl. Phys.* **B 555** (1999) 183 [hep-th/9902197].

[72] A. Karch and E. Katz, *Adding flavor to AdS/CFT*, *JHEP* **06** (2002) 043 [hep-th/0205236].

[73] P. Levai and U.W. Heinz, *Massive gluons and quarks and the equation of state obtained from* SU(3) *lattice QCD*, *Phys. Rev.* **D 57** (1998) 1879 [hep-ph/9710463];
O. Philipsen, *On the non-perturbative gluon mass and heavy quark physics*, *Nucl. Phys.* **B 628** (2002) 167 [hep-lat/0112047];
A. Nakamura, I. Pushkina, T. Saito and S. Sakai, *Screening of hot gluon*, *Phys. Lett.* **B 549** (2002) 133 [hep-lat/0208075];
D. Zwanziger, *Equation of state of gluon plasma from fundamental modular region*, *Phys. Rev. Lett.* **94** (2005) 182301 [hep-ph/0407103];
A. Peshier and W. Cassing, *The hot non-perturbative gluon plasma is an almost ideal colored liquid*, *Phys. Rev. Lett.* **94** (2005) 172301 [hep-ph/0502138].

[74] Y. Imamura, *Supersymmetries and BPS configurations on anti-de Sitter space*, *Nucl. Phys.* **B 537** (1999) 184 [hep-th/9807179].

[75] C.G. Callan Jr. and I.R. Klebanov, *D-brane boundary state dynamics*, *Nucl. Phys.* **B 465** (1996) 473 [hep-th/9511173].

[76] B. Craps, J. Gomis, D. Mateos and A. Van Proeyen, *BPS solutions of a d5-brane world volume in a D3-brane background from superalgebras*, *JHEP* **04** (1999) 004 [hep-th/9901060].

[77] J. Gomis, A.V. Ramallo, J. Simon and P.K. Townsend, *Supersymmetric baryonic branes*, *JHEP* **11** (1999) 019 [hep-th/9907022].

[78] L. Susskind and E. Witten, *The holographic bound in anti-de Sitter space*, hep-th/9805114;
A.W. Peet and J. Polchinski, *UV/IR relations in AdS dynamics*, *Phys. Rev.* **D 59** (1999) 065011 [hep-th/9809022].

[79] A. Güijosa, K. G. Savvidy and O. Tafjord, unpublished.

[80] E. Witten, *Anti-de Sitter space, thermal phase transition and confinement in gauge theories*, *Adv. Theor. Math. Phys.* **2** (1998) 505 [hep-th/9803131].

[81] E. Witten, *Baryons and branes in anti de Sitter space*, *JHEP* **07** (1998) 006 [hep-th/9805112].

[82] M. Shifman, *k strings from various perspectives: QCD, lattices, string theory and toy models*, *Acta Phys. Polon.* **B36** (2005) 3805 [hep-ph/0510098];
P. Giudice, F. Gliozzi and S. Lottini, *Quantum fluctuations of k-strings: a case study*, hep-lat/0609055.

[83] C.P. Herzog and I.R. Klebanov, *On string tensions in supersymmetric* SU(M) *gauge theory*, *Phys. Lett.* **B 526** (2002) 388 [hep-th/0111078];
C.P. Herzog, *String tensions and three dimensional confining gauge theories*, *Phys. Rev.* **D 66** (2002) 065009 [hep-th/0205064];
S.A. Hartnoll and R. Portugues, *Deforming baryons into confining strings*, *Phys. Rev.* **D 70** (2004) 066007 [hep-th/0405214];
Y. Imamura, *Baryon vertices in AdS blackhole backgrounds*, *Prog. Theor. Phys.* **115** (2006) 815 [hep-th/0512314];

JHEP02(2007)084

I.R. Klebanov, J.M. Maldacena and C.B. Thorn, *Dynamics of flux tubes in large-N gauge theories*, *JHEP* **04** (2006) 024 [hep-th/0602255];
H. Firouzjahi, L. Leblond and S.H. Henry Tye, *The* (p, q) *string tension in a warped deformed conifold*, *JHEP* **05** (2006) 047 [hep-th/0603161];
A. Armoni and B. Lucini, *Universality of k-string tensions from holography and the lattice*, *JHEP* **06** (2006) 036 [hep-th/0604055].

[84] R.C. Myers, *Dielectric-branes*, *JHEP* **12** (1999) 022 [hep-th/9910053].

[85] I.W. Taylor and M. Van Raamsdonk, *Multiple Dp-branes in weak background fields*, *Nucl. Phys.* **B 573** (2000) 703 [hep-th/9910052].

[86] A.A. Tseytlin, *On non-abelian generalisation of the Born-Infeld action in string theory*, *Nucl. Phys.* **B 501** (1997) 41 [hep-th/9701125].

[87] H. Dorn, *Nonabelian gauge field dynamics on matrix D-branes*, *Nucl. Phys.* **B 494** (1997) 105 [hep-th/9612120];
C.M. Hull, *Matrix theory, U-duality and toroidal compactifications of M-theory*, *JHEP* **10** (1998) 011 [hep-th/9711179].

[88] P. Bain, *On the non-abelian Born-Infeld action*, hep-th/9909154.

[89] D. d'Enterria, *High* p_T *leading hadron suppression in nuclear collisions at* $s(NN) **(1/2) = 20\,GeV$*–*$200\,GeV$*: data versus parton energy loss models*, *Eur. Phys. J.* **C 43** (2005) 295 [nucl-ex/0504001].

JHEP02(2007)084